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International Baccalaureate®
Baccalauréat International
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MATHEMATICS
STANDARD LEVEL
PAPER 1

Thursday 4 November 2010 (afternoon)

1 hour 30 minutes

Candidate session number

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INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- You are not permitted access to any calculator for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your session number on each answer sheet, and attach them to this examination paper and your cover sheet using the tag provided.
- At the end of the examination, indicate the number of sheets used in the appropriate box on your cover sheet.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.

Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

SECTION A

*Answer **all** the questions in the spaces provided. Working may be continued below the lines, if necessary.*

1. *[Maximum mark: 5]*

The first three terms of an infinite geometric sequence are 32, 16 and 8.

(a) Write down the value of r. *[1 mark]*

(b) Find u_6. *[2 marks]*

(c) Find the sum to infinity of this sequence. *[2 marks]*

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2. *[Maximum mark: 7]*

Let $g(x) = 2x \sin x$.

(a) Find $g'(x)$. *[4 marks]*

(b) Find the gradient of the graph of g at $x = \pi$. *[3 marks]*

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3. *[Maximum mark: 6]*

The diagram shows two concentric circles with centre O.

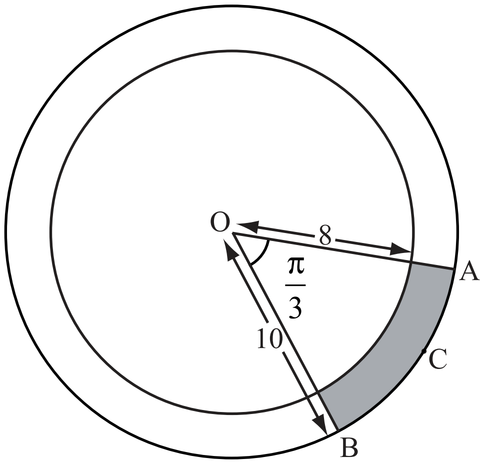

diagram not to scale

The radius of the smaller circle is 8 cm and the radius of the larger circle is 10 cm. Points A, B and C are on the circumference of the larger circle such that $\text{A}\hat{\text{O}}\text{B}$ is $\frac{\pi}{3}$ radians.

(a) Find the length of the arc ACB. *[2 marks]*

(b) Find the area of the shaded region. *[4 marks]*

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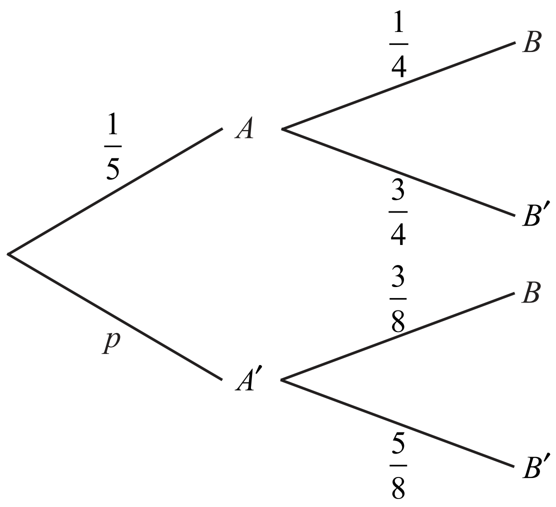
4. *[Maximum mark: 7]*

The diagram below shows the probabilities for events A and B, with $\mathrm{P}(A') = p$.



(a) Write down the value of p. *[1 mark]*

(b) Find $\mathrm{P}(B)$. *[3 marks]*

(c) Find $\mathrm{P}(A'|B)$. *[3 marks]*

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5. *[Maximum mark: 7]*

(a) Show that $4-\cos 2\theta+5\sin\theta=2\sin^2\theta+5\sin\theta+3$. *[2 marks]*

(b) **Hence**, solve the equation $4-\cos 2\theta+5\sin\theta=0$ for $0\le\theta\le 2\pi$. *[5 marks]*

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6. *[Maximum mark: 6]*

The graph of the function $y = f(x)$ passes through the point $\left(\frac{3}{2}, 4\right)$. The gradient function of f is given as $f'(x) = \sin(2x-3)$. Find $f(x)$.

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7. *[Maximum mark: 7]*

Let $\boldsymbol{A} = \begin{pmatrix} 9e^x & e^x \\ e^x & e^{3x} \end{pmatrix}$.

(a) Find an expression for $\det \boldsymbol{A}$. *[2 marks]*

(b) Find the value of x for which $\boldsymbol{A}$ has no inverse. Express your answer in the form $a \ln b$, where $a, b \in \mathbb{Z}$. *[5 marks]*

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*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

SECTION B

*Answer **all** the questions on the answer sheets provided. Please start each question on a new page.*

8. *[Maximum mark: 17]*

The diagram shows quadrilateral ABCD with vertices $A(1, 0)$, $B(1, 5)$, $C(5, 2)$ and $D(4, -1)$.

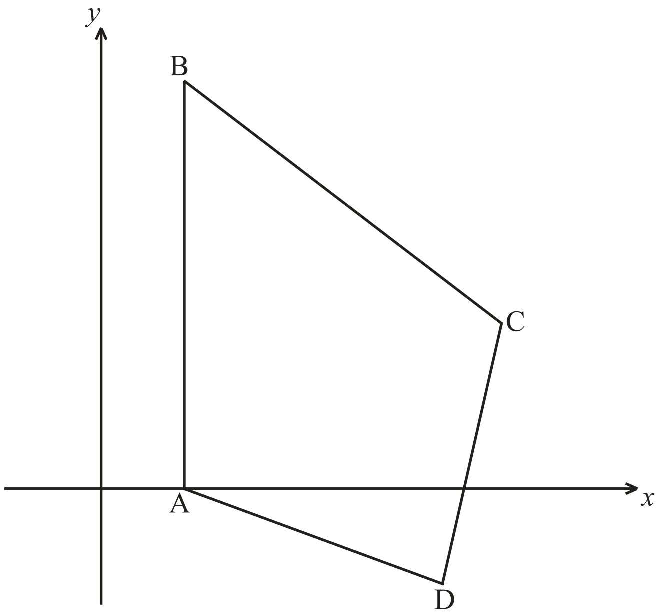


diagram not to scale

(a) (i) Show that $\overrightarrow{AC} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$.

(ii) Find $\overrightarrow{BD}$.

(iii) Show that $\overrightarrow{AC}$ is perpendicular to $\overrightarrow{BD}$. *[5 marks]*

The line (AC) has equation $\boldsymbol{r} = \boldsymbol{u} + s\boldsymbol{v}$.

(b) (i) Write down vector $\boldsymbol{u}$ and vector $\boldsymbol{v}$.

(ii) Find a vector equation for the line (BD). *[4 marks]*

The lines (AC) and (BD) intersect at the point $P(3, k)$.

(c) Show that $k = 1$. *[3 marks]*

(d) **Hence** find the area of triangle ACD. *[5 marks]*

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

9. *[Maximum mark: 12]*

Let $f(x) = x^2 + 4$ and $g(x) = x - 1$.

(a) Find $(f \circ g)(x)$. *[2 marks]*

The vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ translates the graph of $(f \circ g)$ to the graph of h.

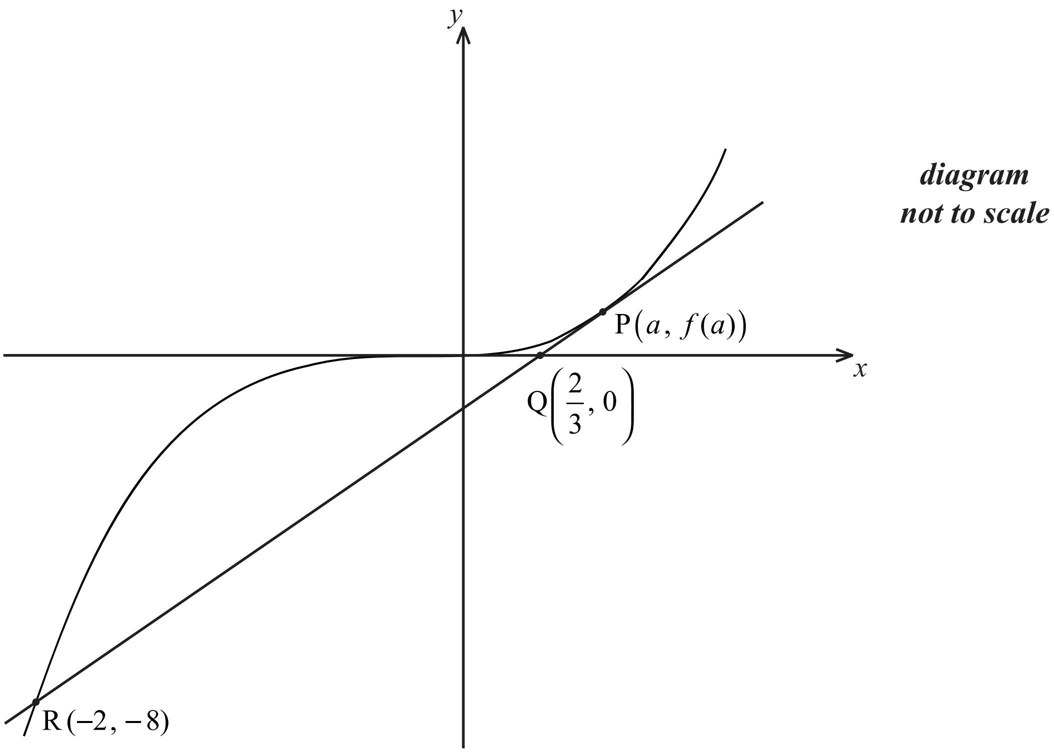
(b) Find the coordinates of the vertex of the graph of h. *[3 marks]*

(c) Show that $h(x) = x^2 - 8x + 19$. *[2 marks]*

(d) The line $y = 2x - 6$ is a tangent to the graph of h at the point P. Find the x-coordinate of P. *[5 marks]*

10. *[Maximum mark: 16]*

Let $f(x) = x^3$. The following diagram shows part of the graph of f.



The point $\mathrm{P}(a, f(a))$, where $a > 0$, lies on the graph of f. The tangent at P crosses the x-axis at the point $\mathrm{Q}\left(\frac{2}{3}, 0\right)$. This tangent intersects the graph of f at the point $\mathrm{R}(-2, -8)$.

(This question continues on the following page)

*Do **NOT** write solutions on this page. Any working on this page will **NOT** be marked.*

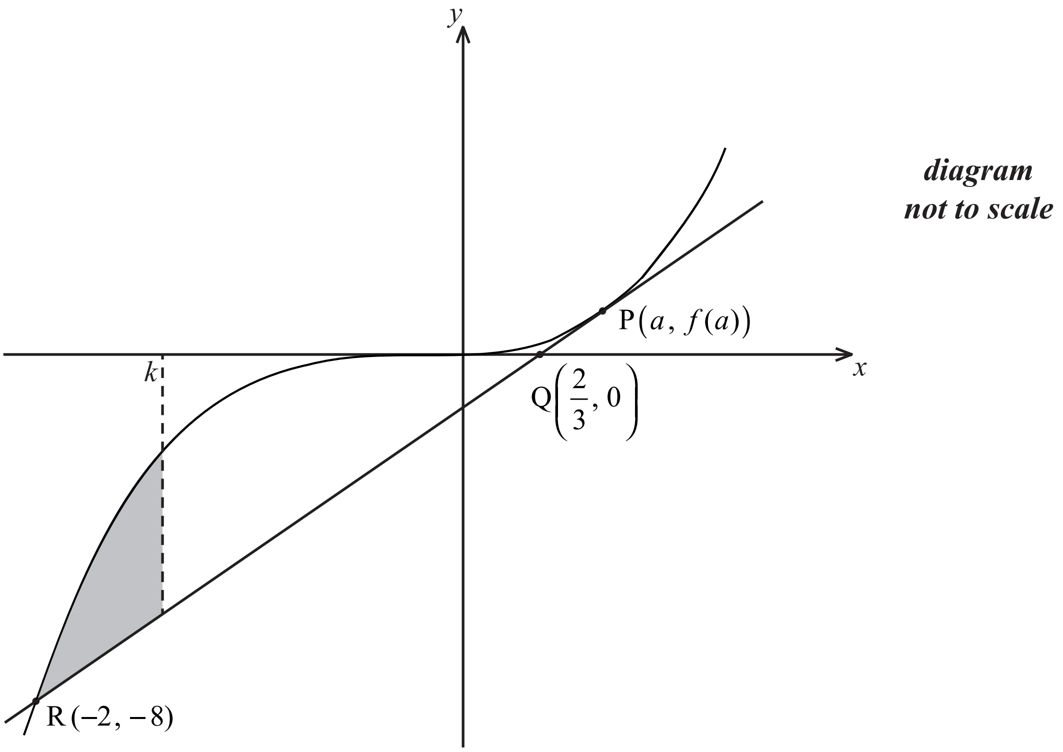
(Question 10 continued)

(a) (i) Show that the gradient of [PQ] is $\dfrac{a^3}{a-\frac{2}{3}}$.

(ii) Find $f'(a)$.

(iii) Hence show that $a=1$. *[7 marks]*

The equation of the tangent at P is $y=3x-2$. Let T be the region enclosed by the graph of f, the tangent [PR] and the line $x=k$, between $x=-2$ and $x=k$ where $-2<k<1$. This is shown in the diagram below.



(b) Given that the area of T is $2k+4$, show that k satisfies the equation $k^4-6k^2+8=0$. *[9 marks]*